Electrostatic fluctuations in collisional plasmas

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We present a theory of electrostatic fluctuations in two-component plasmas where electrons and ions are described by Maxwellian distribution functions at unequal temperatures. Based on the exact solution of the Landau kinetic equation, that includes electron-electron, electron-ion, and ion-ion collision integrals, the dynamic form factor, $S(\vec{k},\omega)$, is derived for weakly coupled plasmas. The collective plasma responses at ion-acoustic, Langmuir, and entropy mode resonances are described for arbitrary wave numbers and frequencies in the entire range of plasma collisionality. The collisionless limit of $S(\vec{k},\omega)$ and the strong-collision result based on the fluctuation-dissipation theorem and classical transport at $T_e = T_i$ are recovered and discussed. Results of several Thomson scattering experiments in the broad range of plasma parameters are described and discussed by means of our theory for $S(\vec{k},\omega)$.

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I. INTRODUCTION

The theory of fluctuations produced by particle discreteness in stable plasmas plays an important role in the statistical description of plasmas [1,2]. These fluctuations are responsible for particle diffusion and transport. Thomson scattering (TS) of electromagnetic radiation from electron-density fluctuations has proven to be a powerful diagnostic in determining plasma parameters and basic plasma processes. Because of the progress that has been achieved in TS experiments in recent years [3,4], the theory of fluctuations remains an active field of research. This article will be concerned with the theory of fluctuations and is motivated by the TS measurements in weakly coupled collisional nonequilibrium plasmas at different electron and ion temperatures, $T_e \neq T_i$. Frequent particle collisions and unequal temperatures characterize dense laser-produced plasmas and high-energy density systems. With the motivation of explaining ionospheric experiments the general form of the Thomson scattered spectrum of a collisionless plasma were determined independently in 1960 by Feyer [5], Renau [6], Daugherty and Farley [7], and Salpeter [8]. These fluctuations caused by particle discreteness have been described by the linearized Vlasov equation, and therefore these theories do not include effects of particle collisions but they work for nonequilibrium plasmas, for example, with $T_e \neq T_i$. Vlasov theory for the dynamical correlation function of density fluctuations will be recovered from our result in the limit of vanishing collisions and for Maxwellian electron and ion particle distribution functions.

The dynamical evolution of the correlation functions of fluctuations is described by the linearized kinetic equations for the one particle distribution function with initial conditions corresponding to static correlations [9,10]. Equivalently, following Onsager's hypothesis [11], fluctuations of dynamical quantities evolve in accordance with the same model equations as those governing macroscopic processes. Thus, the fluctuations on a hydrodynamical scale in thermal equilibrium relax due to collisions according to the equations of linearized hydrodynamics [12,13]. In plasmas the relevant hydrodynamical theories are transport models of Spitzer-Härm [14] and Braginskii [15], which are used to derive collision-dominated fluctuation spectrum. This result will be recovered from our theory in the opposite to Vlasov, strongly collisional limit. The macroscopic model in our theory is the system of linearized equations of nonlocal and nonstationary hydrodynamics that works everywhere from the collisionless Vlasov limit to the strongly collisional limit of hydrodynamical fluctuations. These equations have been derived in the process of finding rigorous solutions to the linearized Vlasov-Landau kinetic equation in the entire regime of plasma collisionality for all frequencies and wave numbers [16–18]. These results are used here in derivation of the correlation functions of density fluctuations. The nonlocal hydrodynamics is formulated in the Fourier space and it is well suited for the problem of finding spectral densities of the correlation functions of fluctuations. In particular, we are interested in finding the electron density correlation function, which is used in the calculation of the TS cross-section.

The nonlocal hydrodynamics has been introduced in our paper because it describes evolution of the fluctuations, and it is used in finding solution to the linearized Vlasov-Landau kinetic equation. One can derive linear plasma response using nonlocal hydrodynamic equations and plasma dispersion function in terms of the nonlocal and nonstationary transport coefficients [17,18]. This method of the solution of the kinetic equation follows the pioneering work by A.R. Bell [19]. He introduced nonlocal thermal conductivity for the first time by considering small amplitude perturbations associated with linear ion acoustic waves. Subsequently, the full set of linear electrostatic modes [17,18,20], including the entropy mode, and the complete linear plasma response in the entire regime of particle collisions have been found. Dispersion and damping of electrostatic modes describe long-time plasma evolution and define resonances of the dynamical form factor discussed in our paper.

We proceed with the solution of the linearized kinetic equations for the one-particle distribution functions. This has been done several times before for the two-component, electron-ion plasmas [21,22] and for the electron transport problem [16,17]. The first moment of the perturbed electron distribution function will be identified with the fluctuation of electron density and used to calculate the density correlation function.

In this study, TS experiments provide the motivation for our work. They have enabled advancements in the understanding of dense plasmas: from the atomic physics of high Z plasmas [23], through nonlocal thermal transport [24] to studies of enhanced fluctuations and plasma turbulence [25,26]. TS is routinely used as a primary diagnostic of plasma parameters such as electron and ion temperatures, particle density and flow velocities [27,28]. And, in recent years, applications of short wavelength laser probes in the x-ray and VUV regimes have made possible for the first time TS experiments in solid density plasmas and shock compressed materials [4].

The wide range of plasma parameters that have been investigated in TS measurements underscores the need for the general expression for the dynamical form factor, $S(\vec{k},\omega)$, such as derived in our paper, to properly account for all k values and frequencies. The utility of Thomson scattering is greatly enhanced when it is used in conjunction with good theoretical models of the dynamical form factor. This paper contains the theory of the dynamical form factor that is valid for all frequencies and k vectors that span the broad regime of parameters from the strongly collisional to the collisionless limits in weakly coupled nonequilibrium plasmas.

The paper is organized as follows: Sec. II summarizes the main steps from the theory of nonlocal and nonstationary hydrodynamics, including initial conditions and source terms, transport relations, and plasma linear dielectric response. The derivation of the dynamical form factor is presented in Sec. III, first for the equilibrium plasma, where $T_e = T_i$, and next for plasmas away from equilibrium, with $T_e \neq T_i$. The accuracy of this derivation is tested by recovering the correct limiting expression of $S(\vec{k},\omega)$ for collisionless plasmas. Applications of our results to different TS experiments are discussed in Sec. IV. These include the strong-collision regime, and low frequency and high frequency limits. Section V discusses the implications of this work and gives a summary of our results.

II. SOLUTION OF THE KINETIC EQUATION

The starting point of our theory of fluctuations is the linearized Vlasov-Landau, or Vlasov-Fokker-Planck, kinetic equation. The stationary background state of the plasma is described by uniform and isotropic Maxwellian distribution functions with different temperatures for electrons and ions. We assume that during the time of correlation evolution the background temperatures remain constant.

A. Linearized Fokker-Planck kinetic equations

The general form of the kinetic equation for the one-particle distribution functions, $F_a(\vec{r},\vec{v},t)$ $(a = e,i)$, in unmagnetized two-component plasmas reads

$$\frac{\partial F_a}{\partial t} + \vec{v}\cdot\vec{\nabla}F_a + \frac{e_a\vec{E}(\vec{r},t)}{m_a}\cdot\frac{\partial F_a}{\partial\vec{v}} = \sum_{r=a,b} C_{ar}(F_a,F_r), \quad (1)$$

where $\vec{E}$ is the self-consistent electrostatic field calculated from Maxwell equations using particle distribution functions. We consider small perturbations of the electron and ion distribution functions. The background plasma state is described by the homogeneous Maxwellians, $F_M^a = n_a(m_a/2\pi T_a)^{3/2}\exp(-m_a v^2/2T_a)$ $(a = e, i)$, including densities n_a $(Zn_i = n_e)$ and temperatures T_a. We will neglect collisional energy equilibration of the background temperatures and keep T_e and T_i constant, $T_e \neq T_i$. This assumption and several approximations below in the collisional operators take advantage of the large mass ratio, $m_i/m_e \gg 1$. The electron and ion distribution functions are represented in terms of background Maxwellians and small perturbations f_a, $F_a = F_M^a + f_a(\vec{r},\vec{v},t)$. The Fourier transformed perturbations of the distribution functions $f_a(v,\mu,t) = \sum_0^\infty f_l^a(v,t)P_l(\mu)$ (we dropped the subscripts $\vec{k}$ indicating Fourier transformed quantities to lighten the notation) are expanded in a series of Legendre polynomials $P_l(\mu)$, where μ is the cosine of the angle between velocity $\vec{v}$ and wave vector $\vec{k}$. With these expansions, the kinetic Eq. (1) with Landau collisional terms are decomposed into two infinite coupled sets of equations for the harmonics $f_l^a(v,t)$ of the electron and ion distribution functions:

$$\frac{\partial f_l^a}{\partial t} + ikv\frac{l}{2l-1}f_{l-1}^a + ikv\frac{l+1}{2l+3}f_{l+1}^a = C_{aa}^l + C_{ab}^l + (e_aE/T_a)vF_M^a\delta_{l1}. \quad (2)$$

The collision operators, C_{ab}^l, are defined by the Rosenbluth potentials. Because of the small mass ratio $m_e/m_i \ll 1$ collisions between electrons and ions can be described in a simplified form [15,17,29,30],

$$C_{ei}^l = -\frac{l(l+1)}{2}\nu_{ei}f_l^e + \delta_{l1}\nu_{ei}\frac{v}{v_{Te}^2}F_M^e u_i,$$

$$C_{ie}^l = \delta_{l1}\left(\frac{v}{v_{Ti}}F_M^i\frac{\mathcal{R}_{ie}}{n_i m_i v_{Ti}} + \nu_{ei}^T\frac{m_e n_e}{m_i n_i}C_{ie}^{(1)}\right),$$

$$C_{ie}^{(1)} = \frac{1}{v^2}\frac{\partial}{\partial v}v^3 f_l^i + \frac{T_e}{m_i v^2}\left(\frac{\partial}{\partial v}v^2\frac{\partial}{\partial v} - l^2 - l\right)f_l^i, \quad (3)$$

where δ_{gh} is the Kronecker δ function, $\nu_{ab}(v) = 4\pi n_b(e_a e_b)^2\Lambda_{ab}/m_a^2 v^3$ is the velocity-dependent particle collision frequencies, Λ_{ab} is the Coulomb logarithm, $u_a = 4\pi\int dv v^3 f_1^a/3n_a$ is the mean particle velocity, $v_{Ta} = \sqrt{T_a/m_a}$ is the particle thermal velocity, $\nu_{ei}^T = \nu_{ei}(v_{Te})\sqrt{2/\pi}/3$, and $\mathcal{R}_{ie} = 4\pi m_e\int dv v^3\nu_{ei}(v)f_1^e/3$ is the friction force. The term $C_{ie}^{(1)}$ in the expression for C_{ie}^l in Eq. (3) is small and can be neglected for the calculation of an ion distribution function. At the same time, $C_{ie}^{(1)}$ is important to ensure ion momentum conservation. For like particle collisions, C_{aa}^l, we use the general form of the collisional operator [22,29,30].

B. Source terms and initial conditions

The handling of source terms in solutions to kinetic Eq. (2) is an important part of the analysis of particle transport in plasmas with arbitrary collisionality [16]. Although transport occurs in response to local gradients in the fluid variables, the generation of such inhomogeneities requires the input of particles or heat into the system. In the strong-collision limit, source terms do not play an important role in the analysis, but when considering arbitrary collision rates, and in particular for the collisionless plasma, they must be treated explicitly as driving terms in the kinetic equations. It was assumed in the derivation of nonlocal transport equations [16] that the sources vanish after some time, $t = 0$, and the nonlocal hydrodynamics properly describe plasma response as the solution to the initial value problem. More importantly, the particular choice of the initial conditions is based on the linearized local Maxwellian distribution functions,

$$f_a(\vec{k},v,t=0) = \left[\frac{\delta n_a(\vec{k},0)}{n_a} + \frac{\delta T_a(\vec{k},0)}{T_a}\left(\frac{v^2}{2v_{Ta}^2} - \frac{3}{2}\right)\right]F_M^a(v). \quad (4)$$

This form of $f_a(t=0)$ enables closure of the hydrodynamic equations by introducing initial perturbations, or sources, in terms of hydrodynamic variables: namely the particle densities $\delta n_a(0)$ and temperatures $\delta T_a(0)$. These initial perturbations are subsequently expressed in terms of perturbations of hydrodynamic variables and fields at later times in the nonlocal transport theory of Refs [16,22,30]. Clearly, the distribution function need not to be Maxwellian Eq. (4) in regime close to $t = 0$ when collisional effects are very weak. On the other hand, it is not clear whether a transport theory can be constructed at all for the fully non-Maxwellian case and whether it would have any advantages compared to a kinetic description.

The role of initial perturbations in the theory of fluctuations, which are studied here is equally important but conceptually much simpler. We are concerned with the state of the plasma, which is well approximated by the local Maxwellian distribution function due to the long-time collisional evolution. Different temperatures of electrons and ions are the only signatures of non-equilibrium effects in the background state of the plasma. Temperatures of the both species are treated as constants because the energy equilibration in the background state is assumed to take place on the time scale much longer than the evolution of fluctuations. However, to describe the evolution of a dynamic form factor we still need to consider the entire range of particle collisionality because the plasma response depends on the product $k\lambda_{ab}$, i.e., the ratio between wavelength of fluctuations $2\pi/k$ and the particle collision mean free path λ_{ab}. As it was the case for nonlocal hydrodynamics [16,22,30], the sources of fluctuations are introduced through the initial conditions Eq. (4) by performing Laplace transformation of Eq. (2),

$$\hat{L}f_l^a - C_{aa}^l - C_{ab}^l = S_l^a,$$
$$\hat{L}f_l^a \equiv -i\omega f_l^a + ikv\frac{l}{2l-1}f_{l-1}^a + ikv\frac{l+1}{2l+3}f_{l+1}^a, \quad (5)$$

where $S_0^a = f_a(t=0)$ [Eq. (4)] and $S_1^a = (e_a E/T_a)vF_M^a$. The higher-order terms S_l^a ($l \geqslant 2$) vanish. We continue with the simplified notation using f_l^a for $f_l^a(k,v,\omega)$.

C. Solutions for the basis functions

We will seek solutions to Eqs. (5) as the linear superposition of different basis functions $\psi_l^{bA}(v)$ ($A = N,T,R$) and ($b = e,i$),

$$f_l^e = \left(i\frac{eE}{kT_e}\delta_{l0} + \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{eE}{kT_e}\right)\psi_l^{eN} + \frac{3}{2}\frac{\delta T(0)}{T_e}\psi_l^{eT} - iku_i\psi_l^{eR}\right)f_M^e, \quad (6)$$

and

$$f_l^i = \left\{i\frac{Zen_iE+\mathcal{R}_{ie}}{kn_iT_i}\delta_{l0} + \left(\frac{\delta n_i(0)}{n_i} + \omega\frac{Zen_iE+\mathcal{R}_{ie}}{kn_iT_i}\right)\psi_l^{iN} + \frac{3}{2}\frac{\delta T_i(0)}{T_i}\psi_l^{iT}\right\}f_M^i, \quad (7)$$

where $\psi_l^{bA}(v)$ describe electron and ion response to the initial fluctuations of particle densities, temperatures, electric field E, ion average velocity u_i and the friction force $\mathcal{R}_{ie}$.

Using relations Eqs. (6) and (7), we can obtain the system of equations for the basis functions $\psi_l^{bA}(v)$ from Eq. (5) with different source terms on the right-hand side,

$$\hat{L}\psi_l^{bA} + \delta_{be}\frac{l(l+1)}{2}\nu_{ei}\psi_l^{bA} - \frac{1}{F_M^b}C_{bb}^l\left[F_M^b\psi_l^{bA}\right] = S_l^{bA}, \quad (8)$$

where δ_{ij} are Kronecker δ functions, $S_l^{bN,bT} = \delta_{l0}S^{bN,bT}$, $S_l^{eR} = 3\delta_{l1}S^R$, and $S^{bN} = 1$, $S^{bT} = v^2/3v_{Tb}^2 - 1$, $S^R = iv\nu_{ei}/3kv_{Te}^2$.

To make further progress we will use a simplified form of C_{aa}^l for $l \gg 1$ in Eq. (8) to close this infinite system of equations. Because of this simplification, starting from the order $l = l_{\max}$, all the equations for the harmonics of the basis functions take on the following simple form ($l > l_{\max}$):

$$2\hat{L}\psi_l^{bA} = -l(l+1)\nu_b^*\psi_l^{bA}, \quad (9)$$

where $\nu_a^* = \nu_{ei}\delta_{ae} + \nu_{aa}(I_0^a + 2J_{-1}^a/3 - I_2^a/3)$, and the two integrals $I_m^a = 4\pi/(n_a v^m)\int_0^v dv v^{m+2}F_M^a$ and $J_m^a = 4\pi/(n_a v^m)\int_v^\infty dv v^{m+2}F_M^a$ have been introduced before, cf., e.g., Ref. [29], p. 276, when evaluating Rosenbluth potentials. The infinite system of Eq. (9) has been solved following the summation procedure [17,31], where one evaluates the renormalized effective collision frequencies ν_l^a from the following recurrence formula:

$$\nu_l^a = -i\omega + \frac{1}{2}l(l+1)\nu_a^* + \frac{(l+1)^2}{4(l+1)^2-1}\frac{k^2v^2}{\nu_{l+1}^a}. \quad (10)$$

Equation (10) can also be represented in terms of continuous fractions. In practice, finding $\nu_{l_{\max}}^a$ with high accuracy requires no more than 20–30 iterations. After that, it is sufficient to solve a finite number of Eq. (8) to find basis functions ψ_l^{bA} for $l \leqslant l_{\max}$ given that $\psi_{l_{\max}+1}^{bA} = i[(l_{\max}+1)/(2l_{\max}+3)](kv/\nu_{l_{\max}+1}^b)\psi_{l_{\max}}^{bA}$. We solve this system of equations, expanding the basis functions ψ_l^{bA} in Sonine-Laguerre polynomials: $\psi_{2l}^{bA} = \frac{\lambda_{bi}}{v_{Tb}}\sum_{n=0}^\infty c_{2l,n}^{bA}L_n^{1/2}(\frac{v^2}{2v_{Tb}^2})$ and $\psi_{2l+1}^{bA} = \frac{\lambda_{bi}v}{v_{Tb}^2}\sum_{n=0}^\infty c_{2l+1,n}^{bA}L_n^{1/2}(\frac{v^2}{2v_{Tb}^2})$, where $\lambda_{ei} = 3\sqrt{\pi/2}v_{Te}/\nu_{ei}(v_{Te}) = v_{Te}/\nu_{ei}^T$ and $\lambda_{aa} = 3\sqrt{\pi}v_{Ta}/\nu_{aa}(v_{Ta}) = v_{Ta}/\nu_{aa}^T$ are the e-i and i-i (e-e) mean free paths. Substitution of this expansion into Eq. (8) gives a system of linear algebraic equations for the coefficients $c_{\ln}^{bA}$. This system was solved with the *Mathematica* software package. The calculations were performed for $l_{\max} = 8$ resulting in an error related to the closure procedure that does not exceed 1–2%.

Now, to exclude initial perturbation from Eqs. (6) and (7), we take the first moments of the kinetic equation to calculate $\delta n_a = 4\pi \int_0^\infty f_0^a v^2 dv$, $\delta T_a = (4\pi m_a/3n_a)\int_0^\infty dv v^2(v^2 - 3v_{Ta}^2) f_0^a$ $(a = e,i)$ and u_i:

$$\frac{\delta n_e}{n_e} = i\frac{eE}{kT_e} + \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{eE}{kT_e}\right) J_N^{eN} + \frac{3}{2}\frac{\delta T_e(0)}{T_e} J_N^{eT} - iku_i J_N^{eR},$$
$$\frac{\delta T_e}{T_e} = \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{eE}{kT_e}\right) J_T^{eN} + \frac{3}{2}\frac{\delta T_e(0)}{T_e} J_T^{eT} - iku_i J_T^{eR}, \tag{11}$$

$$iku_i = \left(\frac{\delta n_i(0)}{n_i} + \omega\frac{Zen_iE + \mathcal{R}_{ie}}{kn_iT_i}\right)\left(1 + i\omega J_N^{iN}\right) + \frac{3}{2}\frac{\delta T_i(0)}{T_i} i\omega J_T^{iN},$$
$$\frac{\delta T_i}{T_i} = \left(\frac{\delta n_i(0)}{n_i} + \omega\frac{Zen_iE + \mathcal{R}_{ie}}{kn_iT_i}\right) J_N^{iT} + \frac{3}{2}\frac{\delta T_i(0)}{T_i} J_T^{iT}, \tag{12}$$

where

$$J_B^{bA} = \frac{4\pi}{n_b}\int_0^\infty v^2 dv \psi_0^{bA} f_M^b S^{bB}, (B = N,T),$$
$$J_R^{eA} = \frac{4\pi}{n_e}\int_0^\infty v^2 dv \psi_1^{eA} f_M^e S^R. \tag{13}$$

Finally, one can derive f_0^a using hydrodynamic moments:

$$f_l^e = i\frac{eE}{kT_e} f_M^e + \left(\frac{\delta n_e}{n_e} - i\frac{eE}{kT_e}\right)\frac{J_T^{eT}\psi_l^{eN} - J_T^{eN}\psi_l^{eT}}{D_{NT}^{eNT}} f_M^e + \frac{\delta T_e}{T_e}\frac{J_N^{eN}\psi_l^{eT} - J_N^{eT}\psi_l^{eN}}{D_{NT}^{eNT}} f_M^e \tag{14}$$
$$- iku_i\left(\psi_l^{eR} - \frac{D_{NT}^{eRT}}{D_{NT}^{eNT}}\psi_l^{eN} - \frac{D_{NT}^{eNR}}{D_{NT}^{eNT}}\psi_l^{eT}\right) f_M^e,$$

$$f_l^i = i\frac{Zen_iE + \mathcal{R}_{ie}}{kn_iT_i} f_M^i \delta_{l0} + iku_i\frac{J_T^{iT}\psi_l^{iN} - J_T^{iN}\psi_l^{iT}}{J_T^{iT} + i\omega D_{NT}^{iNT}} f_M^i + \frac{\delta T_i}{T_i}\frac{\left(1 + i\omega J_N^{iN}\right)\psi_l^{iT} - i\omega J_N^{iT}\psi_l^{iN}}{J_T^{iT} + i\omega D_{NT}^{iNT}} f_M^i, \tag{15}$$

where $D_{AB}^{bCD} = J_A^{bC}J_B^{bD} - J_A^{bD}J_B^{bC}$. We have described above a procedure that allows on reducing kinetic equations to the system of hydrodynamic equations for the first few moments of the distribution functions. This will be further discussed in the next section.

D. Nonlocal and nonstationary hydrodynamics

The first three moments of the kinetic equation give the equations of continuity, motion, and energy balance for electrons $(a = e)$ and ions $(a = i)$. After taking their Fourier transform, linearizing the fluid equations and keeping only components of perturbed vector and tensor quantities along the k vector, we find the following set of hydrodynamic equations in the electrostatic approximation:

$$\frac{\partial \delta n_a}{\partial t} + n_a iku_a = 0,$$
$$\frac{\partial u_a}{\partial t} = \frac{e_a}{m_a}E_a{}^* - \frac{1}{m_an_a}ik\Pi_\parallel^a + \frac{1}{m_an_a}R_{ab}, \tag{16}$$
$$\frac{\partial \delta T_a}{\partial t} + \frac{2}{3n_a}ikq_a + \frac{2}{3}T_a iku_a = 0,$$

where the friction force satisfies $R_{ei} = -R_{ie}$, and $R_{ie} = \mathcal{R}_{ie} - m_e n_e \nu_{ei}^T u_i$, $E_a^* = E - ik(\delta n_a T_a + n_a \delta T_a)/(e_a n_a)$ is the effective electric field, and

$$\Pi_\parallel^a = \frac{8\pi m_a}{15}\int dv v^4 f_2^a, \quad q_a = \frac{2\pi T_a}{3}\int dv v^3\left(\frac{v^2}{v_{Ta}^2} - 5\right) f_1^a \tag{17}$$

are the longitudinal components of the stress tensor, $\Pi_\parallel^a$, and the particle thermal flux, q_a. The following transport relations for electron fluxes were obtained before in Refs. [16,32]:

$$q_e = -\frac{\alpha T_e}{\sigma} j - \kappa_e ik\delta T_e - n_e T_e \beta u_i,$$
$$E_e^* = \frac{j}{\sigma} - \frac{\alpha}{\sigma} ik\delta T_e - \frac{\beta_j}{\sigma} en_e u_i,$$
$$\mathcal{R}_{ie} = -\frac{(1-\beta_j)}{\sigma} enj + \left(\beta + \frac{e\alpha}{\sigma}\right) ikn_e\delta T_e + \left(\frac{e^2 n_e \beta_j(1-\beta_j)}{\sigma} - m_e\beta_r\nu_{ei}^T\right) n_e u_i, \tag{18}$$

where $j = \sum_a e_a n_a u_a$ and the transport coefficients: σ, the electrical conductivity; κ_e, the electron thermal conductivity; α, the thermocurrent coefficient; and different convection transport coefficients, β, β_j, β_r, were introduced before in Refs. [16,32]. They can be expressed in terms of functions J_B^{bA}, J_R^{eA} [Eq. (13)] in the following form:

$$\sigma = \frac{e^2 n_e}{k^2 T_e}\left(\frac{J_T^{eT}}{D_{NT}^{eNT}} + i\omega\right),$$
$$\alpha = -\frac{en_e}{k^2 T_e}\left(\frac{J_T^{eN} + J_T^{eT}}{D_{NT}^{eNT}} + i\omega\right),$$
$$\beta_j = 1 - \frac{D_{NT}^{eRT}}{D_{NT}^{eNT}}, \quad \beta = 1 + \frac{J_T^{eN} - J_T^{eR} + i\omega D_{NT}^{eRN}}{J_T^{eT} + i\omega D_{NT}^{eNT}},$$
$$\beta_r = 1 + \frac{k^2 v_{Te}^2}{\nu_{ei}^T}\left(J_R^{eR} + \frac{J_T^{eR}D_{NT}^{eRN} - J_N^{eR}D_{NT}^{eRT}}{D_{NT}^{eNT}}\right),$$
$$\kappa_a = \frac{n_a}{k^2}\left(\frac{1 + i\omega J_N^{aN}}{J_T^{aT} + i\omega D_{NT}^{aNT}} + \frac{3}{2}i\omega\right), \tag{19}$$

where the thermal transport coefficient, κ_a, is defined for both, electrons, $a = e$, and ions, $a = i$. The transport relations for ion fluxes read [21,22]

$$q_i = -\kappa_i ik\delta T_i - \beta_i n_i T_i u_i, \quad \Pi_\parallel = -4/3ik\eta_i u_i - \beta_i n_i \delta T_i, \tag{20}$$

where β_i and the shear viscosity coefficient η_i are given by the following expressions,

$$\eta_i = \frac{3n_i T_i}{4}\left(\frac{D_{NT}^{iNT}}{J_T^{iT} + i\omega D_{NT}^{iNT}} + \frac{i\omega}{k^2 v_{Ti}^2}\right),$$
$$\beta_i = 1 + \frac{J_T^{iT}}{J_T^{iT} + i\omega D_{NT}^{iNT}}. \tag{21}$$

The transport relations Eqs. (18) and (20) together with nonlocal and nonstationary transport coefficients Eqs. (19) and (21) derived in Refs. [16,21,22,32] provide closure for the fluid Eqs. (16), which are now fully equivalent to the kinetic description in terms of the linearized kinetic Eqs. (2) and (3).

E. Dielectric response

The transport coefficients in Eqs. (18) and (20) and linear response functions below are expressed in terms of time-Fourier transformed quantities. To lighten notation we will simply use the same symbols for functions of frequency as for time dependent quantities. After taking the temporal Fourier transforms of hydrodynamic Eqs. (16), we have eliminated electron temperature perturbation, δT_e, and electron density perturbation, δn_e, from expressions for friction force and electric current:

$$j = \Delta_e\left(-\frac{ie^2 n_e}{k^2 T_e}\omega E + e n_e u_i \Delta_1\right),$$
$$R_{ie} = -e n_e E + \frac{ik^2 T_e}{e\omega}(j\Delta_1 - e n_e u_i \Delta_2), \tag{22}$$

where

$$\Delta_e = \left[1 - i\omega\left(\frac{e^2 n_e}{k^2 T_e \sigma} + \frac{2n_e(\sigma + e\alpha)^2}{\sigma^2(2k^2\kappa - 3i\omega n_e)}\right)\right]^{-1}$$
$$\equiv 1 + i\omega J_N^{eN}, \tag{23}$$

$$\Delta_1 = \left[1 - i\omega\left(\frac{e^2 n_e \beta_j}{k^2 T_e \sigma} + \frac{2n_e(\sigma + e\alpha)(1-\beta)}{\sigma(2k^2\kappa - 3i\omega n_e)}\right)\right],$$
$$\Delta_2 = \left[1 - i\omega\left(\frac{e^2 n_e \beta_j^2}{k^2 T_e \sigma} + \frac{2n_e(1-\beta)^2}{2k^2\kappa - 3i\omega n_e} + \frac{\nu_{ei}^T \beta_r}{k^2 v_{Te}^2}\right)\right]. \tag{24}$$

Similarly, we have eliminated ion temperature perturbation, δT_i, and ion density perturbation δn_i, from Eq. (16) to obtain,

$$R_{ie} + Zen_i E = \frac{ik^2 n_i T_i u_i}{\omega \Delta_i}, \tag{25}$$

where the contribution of the ion transport coefficient is defined by

$$\Delta_i = \left[1 - \frac{\omega^2}{k^2 v_{Ti}^2} - i\omega\left(\frac{4}{3}\frac{\eta_i}{n_i T_i} + \frac{2n_i(1-\beta_i)^2}{2k^2\kappa_i - 3n_i i\omega}\right)\right]^{-1}$$
$$\equiv 1 + i\omega J_N^{iN}, \tag{26}$$

Combining Eqs. (25) and (22), we can express the average ion and electron flow velocities in terms of the electrostatic field:

$$iku_e = -k^2\lambda_{De}^2 \chi_e \frac{eE\omega}{kT_e}, \quad iku_i = k^2\lambda_{De}^2 \chi_i \frac{eE\omega}{kT_e}. \tag{27}$$

We have introduced in Eq. (27) electron, $\chi_e(k,\omega)$, and ion, $\chi_i(k,\omega)$, susceptibility functions,

$$\chi_e = \frac{\Delta_e}{k^2\lambda_{De}^2} + \frac{\Delta_i}{k^2\lambda_{Di}^2}\frac{\Delta_1\Delta_e(\Delta_1\Delta_e - 1)}{1 - g\Delta_i(\Delta_1^2\Delta_e - \Delta_2)} \equiv \frac{1 + i\omega J_N^{eN}}{k^2\lambda_{De}^2}$$
$$+ \frac{1 + i\omega J_N^{iN}}{k^2\lambda_{Di}^2}\frac{(1 + i\omega J_N^{eR})i\omega J_N^{eR}}{1 - ig\omega(1 + i\omega J_N^{iN})\tilde{J}_R^R},$$
$$\chi_i = \frac{\Delta_i}{k^2\lambda_{Di}^2}\frac{\Delta_1\Delta_e}{1 - g\Delta_i(\Delta_1^2\Delta_e - \Delta_2)}$$
$$\equiv \frac{1 + i\omega J_N^{iN}}{k^2\lambda_{Di}^2}\frac{(1 + i\omega J_N^{eR})}{1 - ig\omega(1 + i\omega J_N^{iN})\tilde{J}_R^R}, \tag{28}$$

where $g = ZT_e/T_i$, $\tilde{J}_R^R = J_R^{eR} + \nu_{ei}^T/k^2 v_{Te}^2$, and $\lambda_{Da} = (4\pi e_a^2 n_a/T_a)^{1/2}$ $(a = e,i)$ are the Debye lengths. By utilizing the Maxwell equation $-i\omega E + 4\pi j = 0$, one can define the plasma dielectric function $\epsilon(k,\omega) = 1 + i4\pi j/(\omega E) = 1 + \chi_e + \chi_i$ in the following form:

$$\epsilon = 1 + \frac{\Delta_e}{k^2\lambda_{De}^2} + \frac{\Delta_i}{k^2\lambda_{Di}^2}\frac{\Delta_1^2\Delta_e^2}{1 - g\Delta_i(\Delta_1^2\Delta_e - \Delta_2)}, \tag{29}$$
$$\epsilon = 1 + \frac{1 + i\omega J_N^{eN}}{k^2\lambda_{De}^2} + \frac{1 + i\omega J_N^{iN}}{k^2\lambda_{Di}^2}\frac{(1 + i\omega J_N^{eR})^2}{1 - ig\omega(1 + i\omega J_N^{iN})\tilde{J}_R^R}. \tag{30}$$

Equations (29) and (30) have been recently studied in detail in Ref. [18], wherein solutions for the electrostatic modes, Langmuir and ion-acoustic waves, and also for the entropy mode have been obtained for the entire range of plasma collisionality. These results are important for the understanding of the dynamic form factor and the Thomson scattering cross section.

Before calculating the dynamic form factor we will examine the limiting behavior of $\epsilon(k,\omega)$, in particular, the form of the dielectric function in the absence of collisions. From the solution of the kinetic equations for the basis functions we have $J_N^{eR} = 0$, $\tilde{J}_R^R = 0$ and $J_N^{iN} = iJ_+(\omega/kv_{Ti})/\omega$, $J_N^{eN} = iJ_+(\omega/kv_{Te})/\omega$, where the plasma dispersion function reads

$$W(z) = \frac{1}{\sqrt{2\pi}}\int dv \frac{v}{v - z}\exp(-v^2/2) = 1 - \omega J_+(z),$$
$$z = \omega/kv_{Ta}. \tag{31}$$

Taking the collisionless limit of Eq. (29) and using the above relations one recovers the well-known expression for the plasma dispersion function, $\epsilon = 1 + \sum_{a=e,i} W(\omega/kv_{Ta})/k^2\lambda_{Da}^2$. In the strong collisional limit of $k\lambda_{ai} \to 0$, Eq. (29) assumes a form that is consistent with the results of classical collision-dominated hydrodynamics [15,29].

III. DENSITY FLUCTUATIONS

This section is concerned with the linear plasma response to the initial density fluctuations, which represent external perturbation applied to the plasma. We will derive expressions for the electron density of fluctuations δn_e and the density-density correlation function $G_{ab}(\vec{\rho},\tau) = \langle \delta n_a(\vec{r},t)\delta n_b(\vec{r'},t')\rangle$, where $\vec{\rho} = \vec{r} - \vec{r'}$ and $\tau = t - t'$. Of particular interest will be dynamic form factor $S(k,\omega)$ related to the Fourier-Fourier transform of the electron density correlation function:

$$S(k,\omega) = \frac{\langle \delta n_e^2 \rangle_{k,\omega}}{n_e} = \frac{1}{n_e} \int d^3\rho \int d\tau e^{i\omega\tau - i\vec{k}\cdot\vec{\rho}} G_{ee}(\vec{\rho},\tau). \tag{32}$$

A. Fluctuations in equilibrium plasma

Consider first electron density fluctuations in a complete equilibrium state of a plasma, where $T = T_e = T_i$. In this case, the density fluctuations can be calculated as a full system response to the initial perturbations. On the other hand, in the case of a nonequilibrium plasma with different temperatures of electrons and ions, which is discussed in the next subsection, will require separate treatment of each species in addition to calculations of the self-consistent field from the full system response. The initial perturbations of the plasma electron density are introduced by taking the Laplace transformation of the transport Eqs. (16). In particular,

$$\int_0^\infty dt e^{i\omega t} \frac{\partial \delta n_e}{\partial t} = -i\omega \delta\tilde{n}_e(k,\omega) - \delta n_e(0), \tag{33}$$

where $\delta\tilde{n}_e$ indicates the Lapalce transformed electron density perturbation, $\delta\tilde{n}_e(k,\omega) = \int_0^{+\infty} dt e^{i\omega t} \delta n_e(k,t)$ and $\delta n_e(0) \equiv \delta n_e(k,0)$ is the initial perturbation. It is helpful to distinguish between Laplace transformed perturbations and the Fourier transformed quantities that were used before. Using the Laplace transformed Eqs. (16) one can eliminate temperature perturbations from the expressions for the friction force and the electric current:

$$\tilde{j} = \Delta_e \left[-\frac{ie^2 n_e}{k^2 T}\omega \tilde{E} + e n_e \tilde{u}_i \Delta_1 + \frac{ie}{k}\delta n_e(0) \right],$$
$$\tilde{R}_{ie} = -e n_e \tilde{E} + \frac{ik^2 T}{e\omega}\left[\tilde{j}\Delta_1 - e n_e \tilde{u}_i \Delta_2 - \frac{ie}{k}\delta n_e(0) \right]. \tag{34}$$

Combining Eqs. (25) and (34), we can express the average ion flow velocity in terms of the electrostatic field and the initial density perturbations,

$$ik\tilde{u}_i = k^2 \lambda_{De}^2 \left\{ \chi_i \frac{e\tilde{E}\omega}{kT} - \frac{\delta n_e(0)}{n_e}\chi_C \right\}, \tag{35}$$

where we have introduced a term, χ_C, that is proportional to the electron-ion collision frequency. Here, *e-i* collisions are responsible for coupling of electrons to ion evolution through friction force and ion velocity terms in Eqs. (16),

$$\chi_C = \frac{\Delta_i}{k^2\lambda_{Di}^2} \frac{\Delta_1 \Delta_e - 1}{1 - g\Delta_i\left(\Delta_1^2 \Delta_e - \Delta_2\right)}$$
$$= \frac{1 + i\omega J_N^{iN}}{k^2 \lambda_{Di}^2} \frac{i\omega J_N^{eR}}{1 - ig\omega\left(1 + i\omega J_N^{iN}\right)\tilde{J}_R^R}. \tag{36}$$

Next, we can eliminate the electrostatic field $\tilde{E}$ by means of the Maxwell equation $-i\omega\tilde{E} + 4\pi\tilde{j} = 0$,

$$\tilde{E} = \frac{kT}{e\omega\epsilon(k,\omega)} \frac{\delta n_e(0)}{n_e}\chi_e(\omega,k), \tag{37}$$

where we used the following relations:

$$\epsilon = 1 + \frac{\Delta_e}{k^2\lambda_D^2} + \chi_i \Delta_1 \Delta_e,$$
$$\chi_e = \frac{\Delta_e}{k^2\lambda_D^2} + \chi_C \Delta_1 \Delta_e, \tag{38}$$

which follow from Eqs. (28), (29), and (36). Finally, we substitute the expressions for $\tilde{j}$ [Eq. (34)], $\tilde{u}_i$ [Eq. (35)], and $\tilde{E}$ [Eq. (37)] into the first equation of the system Eq. (16) for $\delta\tilde{n}_e$ and write the Fourier-Laplace transformed electron density fluctuation in terms of initial values $\delta n_e(k,0)$,

$$\delta\tilde{n}_e(\omega,k) = \frac{i}{\omega}\left\{ 1 - k^2\lambda_{De}^2 \frac{\chi_e(1+\chi_i)}{\epsilon} + k^2 \lambda_{De}^2 \chi_C \right\} \delta n_e(0). \tag{39}$$

The relation Eq. (39) for $\delta\tilde{n}_e(\omega,k)$ has been derived from the solutions to the kinetic equation in terms of Laplace transformed quantities, which evolve in response to the initial perturbation, $\delta n_a(k,0)$. Using these quantities we have to define the pair correlation function in the Fourier space $\langle \delta n_a \delta n_b \rangle_{k,\omega}$ as follows:

$$\langle \delta n_a \delta n_b \rangle_{k,\omega} = \mathrm{Re}[\langle \delta\tilde{n}_a(k,\omega)\delta n_b(-k,0)\rangle$$
$$+ \langle \delta\tilde{n}_b(k,\omega)\delta n_a(-k,0)\rangle]. \tag{40}$$

For the initial density correlation function, we will assume a simple equilibrium plasma result consistent with the weakly coupled limit, $G_{ab}(k,0) = \langle \delta n_a(k,0)\delta n_b(-k,0)\rangle = n_b \delta_{ab}$. This leads to the following expression for the Fourier-Fourier transformed electron density correlation function for the density fluctuations about the complete equilibrium state ($T_e = T_i$),

$$\langle \delta n_e^2 \rangle_{\omega,k} = \frac{2k^2\lambda_{De}^2 n_e}{\omega} \mathrm{Im}\left[\frac{\chi_e(1+\chi_i)}{\epsilon} - \chi_C \right]. \tag{41}$$

We can recover from Eq. (41) the well-known limit of the collisionless plasma where the susceptibility functions χ_e, χ_i [Eq. (28)] are evaluated using Eq. (31) as discussed at the end of Sec. II and the coupling due to electron-ion collisions is neglected, $\chi_C = 0$. Validity of Eq. (41) in the strong-collision limit will be discussed in Sec. IV where we will compare Eq. (41) with the results of the fluctuation-dissipation theorem [33] and the full set of classical hydrodynamic equations describing fluctuations. One can apply the fluctuation-dissipation theorem because we have dealt so far with complete plasma equilibrium conditions ($T_e = T_i$). Such calculations and experimental results were discussed in Refs. [34] where dissipation was described in the strongly collisional limit using Braginskii's transport equations [15].

B. Fluctuations in two-temperature plasma

The same procedure of Sec. III A when it is formally applied to nonequilibrium plasmas with two different background temperatures, $T_e \neq T_i$, leads to incorrect results for the density correlation function. This can easily be seen in the collisionless limit of Eq. (41) where there is an extra factor of T_i/T_e multiplying ionic contributions (cf., e.g., Ref. [3] and Eq. (5.6.4) therein) for $T_e \neq T_i$. Difficulties in calculating fluctuations in nonequilibrium plasmas are evident when the fluctuation-dissipation theorem [33] is used and they have been discussed early on (cf., e.g., Ref. [35]). More recently, kinetic theory based on the Klimontovich formalism with stochastic force sources has been applied in the derivation of the dynamic form factor in collisional nonequilibrium plasmas [36]. The generalized fluctuation-dissipation theorem of Ref. [36] led to a dynamic form factor of a similar form as in our theory for the nonequilibrium stationary plasmas [Eq. (51)]. However, the rigorous solution to the full set of linearized Vlasov-Fokker-Planck kinetic equations, without simplifying assumptions, remains the unique feature of our theory. To describe the evolution of fluctuations in the two subsystems that correspond to the two species plasma we have utilized the set of nonlocal and nonstationary transport equations that are fully equivalent to the linearized kinetic equations. These two components of the nonequilibrium plasma are coupled by the self-consistent electric field. To start the calculations we first ignore this field and calculate the spontaneous fluctuations and linear response functions associated with each species. This will be achieved by introducing generalized test particle fluctuations, δn_e^t, δn_i^t, which are not affected by the self-consistent field and polarization of the plasma. The dynamics of these test particle fluctuations is governed by convective and pressure terms and by particle collisions. They also depend on the temperature of each species. In the second stage, we will calculate the fluctuations and linear response functions for the whole electron-ion plasma with self-consistent electric fields in response to test particle fluctuations, δn_e^t, δn_i^t.

1. Initial value problem for the calculation of test particle fluctuations

As before in Sec. III A, we will apply the Laplace transformation to Eqs. (16) and introduce the initial conditions for the density perturbations, $\delta n_a(0)$, except that now the self-consistent field, $E = 0$, and density perturbations correspond to test particles. We will eliminate electron temperature perturbation from the expressions of the friction force and the electric current in the electron formalism,

$$\tilde{j} = \Delta_e\left(en_e\tilde{u}_i\Delta_1 + \frac{ie}{k}\delta n_e(0)\right),$$
$$\tilde{R}_{ie} = \frac{ik^2T_e}{e\omega}\left(\tilde{j}\Delta_1 - en_e\tilde{u}_i\Delta_2 - \frac{ie}{k}\delta n_e(0)\right), \tag{42}$$

and ion temperature perturbations in the formulas relevant to the ion response,

$$\tilde{R}_{ie} + \frac{kT_i}{\omega}\delta n_i(0) = \frac{ik^2n_iT_i\tilde{u}_i}{\omega\Delta_i}. \tag{43}$$

Combining Eqs. (43) and (42) we can express the average ion flow velocity in terms of the initial density perturbations,

$$ik\tilde{u}_i = k^2\lambda_{De}^2\left\{-\frac{\delta n_e(0)}{n_e}\chi_C + \frac{T_i}{T_e}\frac{\delta n_i(0)}{n_e}(\chi_i - \chi_C)\right\}. \tag{44}$$

Finally, we substitute our expressions for $\tilde{u}_i$ [Eq. (44)] and $\tilde{j}$ [Eq. (42)] into the first equation of system [Eqs. (16)] for $\delta\tilde{n}_a^t$ and write the Fourier-Laplace transformed density fluctuation in terms of initial values $\delta n_a(k,0)$,

$$\delta\tilde{n}_e^t(k,\omega) = \frac{i}{\omega}\left\{\left(1 - k^2\lambda_{De}^2(\chi_e - \chi_C)\right)\delta n_e(0) + \frac{T_i}{T_e}k^2\lambda_{De}^2\chi_C\delta n_i(0)\right\}, \tag{45}$$
$$\delta\tilde{n}_i^t(k,\omega) = \frac{i}{\omega}\left\{\left(1 - \frac{T_i}{ZT_e}k^2\lambda_{De}^2(\chi_i - \chi_C)\right)\delta n_i(0) + \frac{k^2\lambda_{De}^2}{Z}\chi_C\delta n_e(0)\right\}. \tag{46}$$

We proceed by calculating the density fluctuations produced by test particle perturbations when the coupling between the two species by the self-consistent field is restored.

2. Response to test particle fluctuations

Density perturbations due to test particles are related to the external source current $j^t = e\omega(Z\delta n_i^t - \delta n_e^t)/k$, which is included into the Maxwell equation $4\pi(j + j^t) - i\omega E = 0$. Note that all perturbations are now Fourier-Fourier transformed. Then, one can proceed and calculate electrostatic self-consistent field due to test density perturbation as follows:

$$E = i\frac{4\pi e}{k\epsilon}\left(\delta n_e^t - Z\delta n_i^t\right). \tag{47}$$

The electrostatic field Eq. (47) is related to density fluctuations, which are calculated by using continuity equations and our relations Eq. (27),

$$\delta n_e(k,\omega) = \delta n_e^t + \chi_e\frac{ikE}{4\pi e} = \frac{1+\chi_i}{\epsilon}\delta n_e^t + \frac{\chi_e}{\epsilon}Z\delta n_i^t,$$
$$\delta n_i(k,\omega) = \delta n_i^t - \chi_i\frac{ikn_iE}{4\pi en_e} = \frac{1+\chi_e}{\epsilon}\delta n_i^t + \frac{\chi_i}{Z\epsilon}\delta n_e^t. \tag{48}$$

Using these expressions for the density fluctuations, we can calculate the electron density correlations in terms of test particles densities (cf Ref. [36]):

$$\langle\delta n_e\delta n_e\rangle_{k,\omega} = \frac{|1+\chi_i|^2}{|\epsilon|^2}\langle\delta n_e^t\delta n_e^t\rangle_{k,\omega} + Z^2\frac{|\chi_e|^2}{|\epsilon|^2}\langle\delta n_i^t\delta n_i^t\rangle_{k,\omega} + Z\frac{(1+\chi_i)\chi_e^*}{|\epsilon|^2}\langle\delta n_e^t\delta n_i^t\rangle_{k,\omega} + Z\frac{(1+\chi_i^*)\chi_e}{|\epsilon|^2}\langle\delta n_i^t\delta n_e^t\rangle_{k,\omega}. \tag{49}$$

In the remaining theory we will use $\delta\tilde{n}_a^t$ Eqs. (45) and (46) to obtain the test particle correlation functions, and after substituting them into Eq. (49) we will find the dynamical form factor for nonequilibrium plasmas.

3. The dynamic form factor

Relations Eqs. (45) and (46) for $\delta n_a^t(k,\omega)$ $(a = e,i)$ have been derived from the solutions to the kinetic equation in terms of Laplace transformed quantities Eq. (33). They can be used to construct the Fourier transformed correlation function in accordance with Eq. (40). By using Eqs. (45) and (40), we calculate the test density correlation functions:

$$\langle \delta n_e^t \delta n_e^t \rangle_{k,\omega} = \frac{2n_e k^2 \lambda_{De}^2}{\omega} \mathrm{Im}(\chi_e - \chi_c),$$
$$\langle \delta n_i^t \delta n_i^t \rangle_{k,\omega} = \frac{2n_e k^2 \lambda_{De}^2}{Z^2 \omega} \frac{T_i}{T_e} \mathrm{Im}(\chi_i - \chi_c),$$
$$\langle \delta n_e^t \delta n_i^t \rangle_{k,\omega} = -\frac{2n_e k^2 \lambda_{De}^2}{Z \omega} \left(1 + \frac{T_i}{T_e}\right) \mathrm{Im}(\chi_c). \tag{50}$$

Substituting correlation function of test particle fluctuations Eq. (50) into Eqs. (49) for the electron-electron correlation functions we obtain the following expression for the dynamic form factor $S(k,\omega) = \langle \delta n_e^2 \rangle_{k,\omega} / n_e$:

$$S(k,\omega) = \frac{2k^2 \lambda_{De}^2}{\omega |\epsilon|^2} \left\{ |1 + \chi_i|^2 \mathrm{Im}[\chi_e - \chi_C] + \frac{T_i}{T_e} |\chi_e|^2 \mathrm{Im}[\chi_i - \chi_C] - \left(1 + \frac{T_i}{T_e}\right) \mathrm{Re}[(1 + \chi_i)\chi_e^*] \mathrm{Im}[\chi_C] \right\}. \tag{51}$$

Equation (51) is the main result of our theory. In equilibrium plasma, where $T_e = T_i$ Eq. (51) has the form identical to Eq. (41). $S(k,\omega)$ Eq. (51) is valid in the entire regime of particle collisionality, $0 \leqslant k\lambda_{\alpha\beta} \leqslant \infty$, in weakly coupled plasmas and it accounts for the collective plasma response in terms of Langmuir, ion-acoustic, and entropy wave resonances. Our result is a generalization of the theory from Ref. [37]. As compared to previous studies, $S(k,\omega)$ Eq. (51) includes the high-frequency response, entropy waves, and charge separation effects and has been derived from a complete solution of the kinetic equation [18] without simplifying assumptions about the plasma parameters. For example, restrictions to $Z \gg 1$ are not necessary. In fact, it is for the first time that $S(k,\omega)$ Eq. (51) has been derived in a form that allows applications to weakly coupled plasmas at all k vectors and frequencies. We can obtain the results with arbitrary accuracy, including for nonequilibrium plasmas where $T_e \neq T_i$. We will describe applications of our theory in unmagnetized plasmas with emphasis on laser produced plasmas where Thomson scattering has become one of the most important diagnostic technique.

In the collisionless limit of $k\lambda_{\alpha\beta} \gg 1$, the dynamic form factor $S(k,\omega)$ Eq. (51) takes the form of the well known expression [3], which was derived for the first time in Refs. [5–8], and it is equivalent to the classical limit of the random phase approximation (RPA) expression [4]. The collisionless limit of Eq. (51) can be achieved using definitions from Eq. (31) and $J_A^R = 0$, $J_i = iJ_+(\omega/kv_{Ti})/\omega$, $J_N^N = iJ_+(\omega/kv_{Te})/\omega$. After introducing $\chi_\alpha^V = W(\omega/kv_{Ta})/k^2\lambda_{Da}^2$, we can show that $S(k,\omega)$ Eq. (51) leads to

$$S^V(\vec{k},\omega) = \frac{\sqrt{2\pi}}{k} \frac{\left[\left(\frac{1}{v_{Te}}\right)\exp\left(-\frac{\omega^2}{2k^2 v_{Te}^2}\right)\left|1 + \chi_i^V(\vec{k},\omega)\right|^2 + \left(\frac{1}{v_{Ti}}\right)\exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right)|\chi_e^V(\vec{k},\omega)|^2\right]}{\left|1 + \chi_e^V(\vec{k},\omega) + \chi_i^V(\vec{k},\omega)\right|^2}, \tag{52}$$

where the superscript V indicates collisionless dynamical evolution of correlations that is described by the Vlasov equation. The rest of this paper will examine effects of particle collisions on the high frequency Langmuir wave spectra, low-frequency ion acoustic and entropy fluctuations using $S(\vec{k},\omega)$ Eq. (51). We will compare Eq. (51) with results of the theory of hydrodynamic fluctuations based on the Braginskii's model [15] and for the high-frequency plasma fluctuations we will also discuss Born-Mermin (BM) theory [38,39] of plasma response. Comparison with the BM approximation will help to define limits of applicability of our theory in dense plasmas approaching strongly coupled regime.

IV. RESULTS AND APPLICATIONS

The main application for the theory of the dynamic form factor is in the calculation of the Thomson scattering cross section [3]. In TS experiments, the $\vec{k}$ vector is defined by the geometry of the scattering process, $\vec{k} = \vec{k}_1 - \vec{k}_0$ (satisfying $\omega = \omega_1 - \omega_0$), where $k_{0,1} = 2\pi/\lambda_{0,1}$ and $\omega_{0,1}$ are the wave number and frequency of the pump (0) and scattered (1) light waves. In experiments, the angle θ between $\vec{k}_0$ and $\vec{k}_1$ is typically fixed, but the magnitude of k_1 is changed as different frequencies (wavelengths λ_1) are examined in the scattered light spectrum. It is customary to plot $S(\vec{k},\omega)$ as a function of λ_1, and this will be done in the following.

A. Limit of strong collisions

As promised in Sec. III A we will first compare our theory with earlier results [34], which were obtained using the fluctuation-dissipation theorem ($T_e = T_i = T$) and Braginskii transport equations [15] describing dissipation in the plasma in terms of collisional transport coefficients. These papers [34] included experimental results from TS experiments with a CO_2 laser probe that were well reproduced by the theoretical form factor. Some discrepancies are always expected as the Braginskii hydrodynamics and transport relations are valid in the $k\lambda_{ab} \ll 1$ limit, where k is inverse of the gradient scale length and λ_{ab} are collisional mean-free-paths. The dynamic form factor was defined via the fluctuation-dissipation theorem [2,33] in the following form [40]:

$$S(k,\omega) = \frac{k^2 T}{\pi \omega^2 e^2 n_e} \mathrm{Re}[\sigma_e(k,\omega)], \tag{53}$$

where σ_e is the AC electric collisional conductivity that describes the electron current responding to external electric

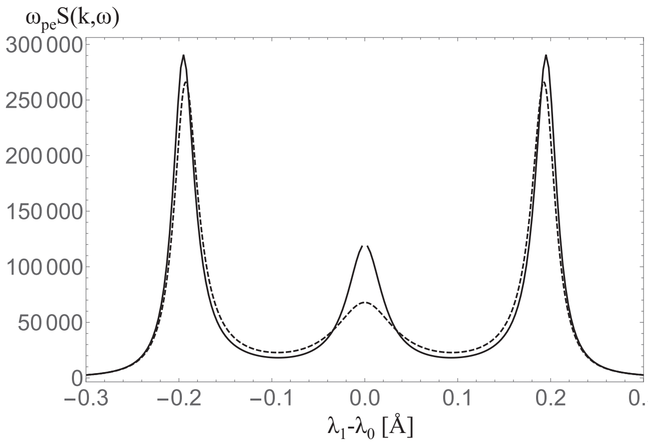

FIG. 1. Dynamical form factors for argon plasma at $n_e = 10^{17}$ cm^{-3}, $T = 2$ eV, $Z = 1$, $A = 18$. The probe wavelength is $\lambda_0 = 10.6$ μm and the scattering angle $\theta = 6^\circ$. Dashed line is obtained using Eq. (54) and the continuous black line corresponds to the full theoretical $S(k,\omega)$ of our theory Eq. (51) for $T_e = T_i = T$.

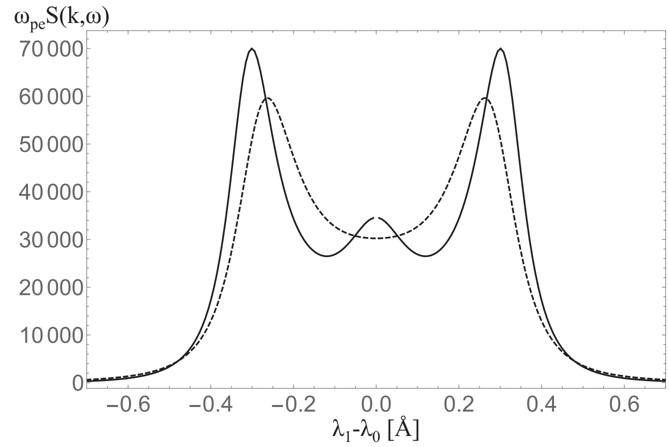

FIG. 2. Dynamical form factors for argon plasma at $n_e = 10^{17}$ cm^{-3}, $T = 5$ eV, $Z = 1$, $A = 18$. The probe wavelength is $\lambda_0 = 10.6$ μm and the scattering angle $\theta = 6^\circ$. Dashed line is obtained using Eq. (54) and the continuous black line corresponds to the full theoretical $S(k,\omega)$ of our theory Eq. (51) for $T_e = T_i = T$.

field perturbation acting on electrons. Braginskii's equations [15] that are used to evaluate σ_e were simplified and only three dominant transport processes were retained described by the electron thermal conductivity $\kappa_{e0} = 3.14 n_e v_{Te}^2/\nu_{ei}$, the ion thermal conductivity $\kappa_{i0} = 3.91 n_i v_{Ti}^2/\nu_{ii}$, and the ion viscosity $\eta_{i0} = 0.96 n_i T_i/\nu_{ii}$. All three transport coefficients represent stationary ($\omega \to 0$) and local ($k \to 0$) limits of the transport coefficients that were introduced in transport relations Eqs. (18) and (20) of our theory for plasmas with $Z = 1$ according to the experimental conditions in Ref. [34]. With these approximations, the dynamical form factor Eq. (53) has the following form [34]:

$$
\begin{aligned}
S(k,\omega) &= 2\frac{A(k) + B(k)b(k)/D(k,\omega)}{[A(k) + B(k)b(k)/D(k,\omega)]^2\omega^2 + H(k,\omega)^2},\\
H(k,\omega) &= 2 - \omega^2/\omega_{0i}^2 + 1.5B(k)\omega^2/D(k,\omega),\\
B(k) &= 1 + 3(m_e/m_i)n_e\nu_{ei}/(k^2\kappa_{e0}),\\
A(k) &= n_e/(k^2\kappa_{e0}) + (4/3)\eta_{i0}/(n_e T),\\
D(k,\omega) &= (3\omega/2)^2 + b(k)^2,\ \omega_{0i} = k(T/m_i)^{1/2},\\
b(k) &= k^2\kappa_{i0}/n_e + 3(m_e/m_i)\nu_{ei}.
\end{aligned} \tag{54}
$$

Comparison between results for the dynamic form factor based on Eq. (54) and our theory Eq. (51) is shown in Fig. 1. At the typical plasma parameters from Ref. [34] used in Fig. 1 we find that the collisional parameter for electrons is $k\lambda_{ei} = 0.08$ and for ions $k\lambda_{ii} = 0.11$. The TS parameter $\alpha = 1/(k\lambda_{De}) = 489.7$. While for these parameters electrons can be described by the Braginskii transport theory, i.e., nonlocal effects are small, the ion response will nevertheless be affected by nonlocal effects. In particular, the ion thermal conductivity is reduced from the κ_{i0} and this lowers the damping of the entropy mode as it is seen in Fig. 4 of Ref. [18], cf. also Ref. [20]. This explains the discrepancy between two curves in Fig. 1 at the zero-frequency entropy mode.

This trend continues for plasma parameters corresponding to less collisional plasmas. Figure 2 shows the results for increased temperature of $T_e = T_i = 5$ eV (the rest of parameters are as in Fig. 1). In hotter plasmas, collisional parameters are $k\lambda_{ei} = 0.38$ and for ions $k\lambda_{ii} = 0.54$ and the TS parameter is $\alpha = 1/(k\lambda_{De}) = 309.7$.

Now, the strong collision theory Eq. (54) is not only incorrect for the entropy mode but electron transport is also in the nonlocal regime causing changes in the frequency and damping of the ion-acoustic fluctuations. Again, results in Fig. 2 reflect changes to the dispersion relations of the ion-acoustic waves and entropy modes in the regime of weaker collisions discussed in Ref. [18].

B. Low-frequency fluctuations

TS in the collective regime ($\alpha = (k\lambda_{De})^{-1} > 1$) and in the low frequency range ($\omega \leqslant \omega_{pi}$) is used to investigate ion-acoustic and entropy mode fluctuations. For $\alpha \gg 1$ the dynamic form factor $S(\vec{k},\omega)$ characterizes long wavelength fluctuations in the hydrodynamical regime as discussed in the previous section. However, for the typical conditions in laser-produced plasmas, $S(\vec{k},\omega)$ will be in the weakly collisional regime where damping and dispersion of the modes depend on the nonlocal and nonstationary properties of transport relations. To illustrate these features of the $S(\vec{k},\omega)$ theory, we will first address a typical regime encountered in carbon plasmas that is characteristic of laser produced plasmas at modest intensities. In fact, we will discuss results relevant to measurements in Ref. [27] (cf. Fig. 4 therein). Consider the TS probe at $\lambda_0 = 5270$ Å, a scattering angle of $\theta = 117^\circ$, with $T_e = 100$ eV, $n_e = 5.6\ 10^{18}$ cm^{-3}, and $\alpha = 1.58$ in carbon plasmas. At these conditions $k\lambda_{ei} = 133$ and therefore electron collisions have no effect on the TS cross-section. On the other hand, for the three ion temperatures examined in Ref. [27] ion-ion collisions play a role and their effects are illustrated in Fig. 3.

Figure 3 displays three panels (a), (b), and (c) that show redshifted ion-acoustic peaks calculated for the above plasma parameters and for different ion temperatures using

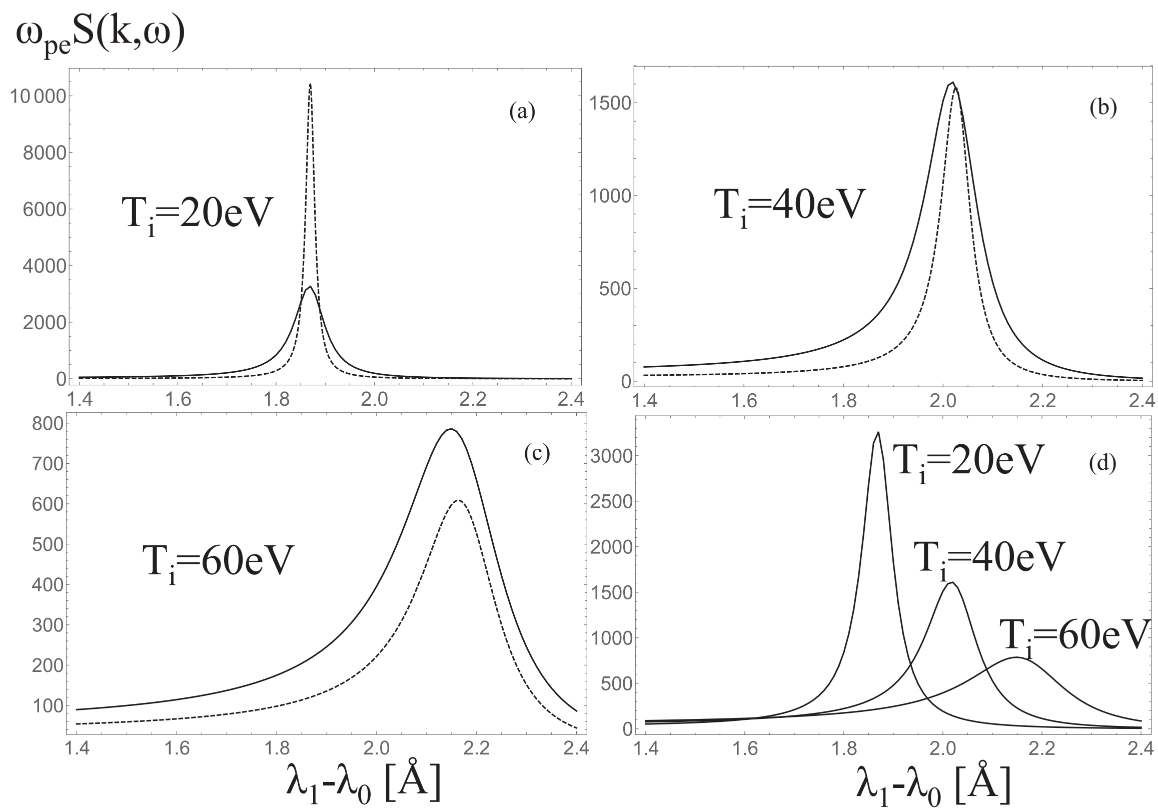

FIG. 3. Dynamic form factors for Thomson scattering probe at $\lambda_0 = 5270$ Å, scattering angle $\theta = 117^\circ$, $T_e = 100$ eV, $n_e = 5.6\ 10^{18}$ cm^{-3}, and $\alpha = 1.58$. Only red shifted ion acoustic peaks are shown for three different ion temperatures in panels (a), (b), and (c). Results of the full collisional theory Eq. (51) of $S(\vec{k},\omega)$ are depicted by continuous lines, the collisionless $S^V(\vec{k},\omega)$ expression Eq. (52) gives results depicted by dashed lines. Panel (d) shows all three cases calculated using Eq. (51).

our complete theory for $S(\vec{k},\omega)$ Eq. (51) (solid lines) and collisionless $S^V(\vec{k},\omega)$ theory Eq. (52) (dashed lines). The latter was also employed in Ref. [27]. The two ion acoustic peaks are symmetric and there is no entropy mode perturbation thus by showing only one peak we fully illustrate TS results. Figure 3(d) combines all three cases in one plot for the collisional plasma calculations. The origin of discrepancies between two theories are ion collisions. The most significant effect is for $T_i = 20$ eV in Fig. 3(a) resulting in $k\lambda_{ii} = 0.68$, i.e., relatively collisional case where ion collisions play the most important role within these three examples. At the same time, $ZT_e/T_i = 30$ ($Z = 6$) is the largest in Fig. 3(a) and

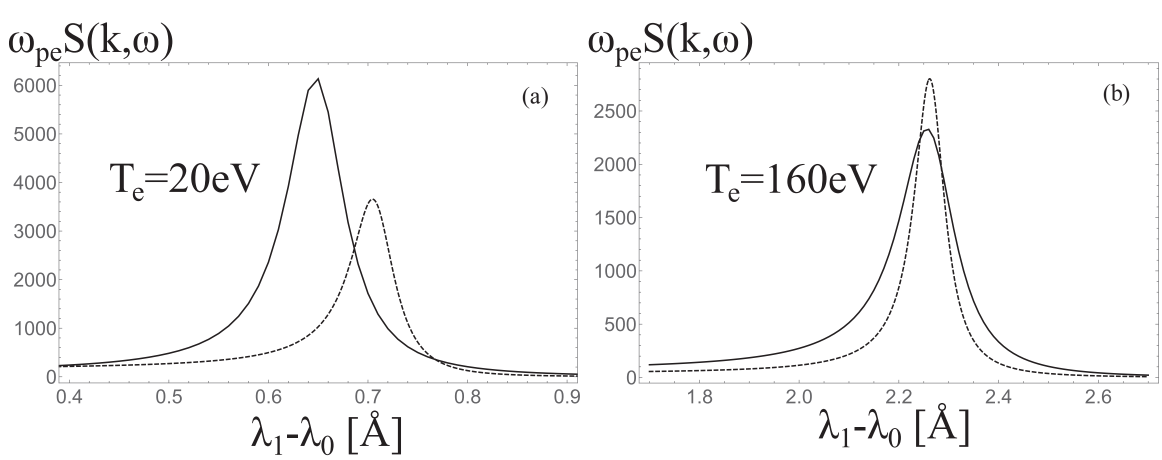

FIG. 4. Dynamic form factors for $n_e = 10^{19}$ cm^{-3}, $\lambda_0 = 5320$, $\theta = 90^\circ$ in nitrogen plasma. Only one, red shifted peak is shown out of two symmetric resonances. Results for collisional plasma with $k\lambda_{ei} = 5.67$, $k\lambda_{ii} = 0.27$, $Z = 4$, $T_e = 2T_i = 20$ eV are shown in (a), results for $T_e = 2T_i = 160$ eV, $Z = 7$, $k\lambda_{ei} = 129$, $k\lambda_{ii} = 2.23$ are shown in panel (b). Dashed lines correspond to the collisionless theory Eq. (52), continuous lines correspond to the complete theory of the dynamical form factor Eq. (51).

therefore ion Landau damping is negligible. This narrows the width of the ion acoustic peak for $S^V(\vec{k},\omega)$ Eq. (52) results. The intermediate case in Fig. 3(b) for $T_i = 40$ eV, $k\lambda_{ii} = 2.2$, shows enough broadening of the ion-acoustic peak due to collisions that the result may exceed the accuracy of the experimental spectra fitting in Ref. [27]. At $T_i = 60$ eV, $k\lambda_{ii} = 4.0$, the two theories start to converge as the plasma becomes less collisional. The importance of ion collisions increases in high $- Z$ plasmas. Thus, TS from gold plasmas, particularly for ICF related conditions and very high temperatures is dominated by ion collisions at $Z = 40$–50.

TS from the nitrogen gas jet plasma in Ref. [24] was used to map out temperature profiles of the propagating heat waves and led to one of the most convincing demonstration of the nonlocal heat conduction in laser produced plasmas. Dynamical form factors in this experiment were calculated over a broad range of electron temperatures from $T_e = 20$ eV, cf. Fig. 4(a), up to $T_e = 160$ eV, cf. Fig. 4(b) at electron density $n_e = 10^{19}$ cm^{-3}. The electron temperature was deduced in [24] from the peak separation in the ion acoustic wave spectra and discrepancy shown in Fig. 4(a) between the correct collisional theory and collisionless results used in Ref. [24] is well within experimental accuracy of these measurements. The comparisons between collisionless and collisional theory in Fig. 4(a) involves k vectors such that $k\lambda_{ei} = 5.67$ and $k\lambda_{ii} = 0.27$. The shift in the position of the ion-acoustic peak is consistent with the effect of collisions on the ion acoustic frequency [18]. With the parameters of Fig. 4(a) also electrons in addition to ions (as in Fig. 3) contribute to damping and dispersion of the ion acoustic waves through nonlocal and nonstationary transport coefficients. Contributions of ions remain in the weakly collisional regime at higher temperatures ($T_e = 2T_i = 160$ eV) and for $Z = 7$ in Fig. 4(b), where $k\lambda_{ii} = 2.23$. While the electron's contributions are in collisionless regime, $k\lambda_{ei} = 129$.

C. High-frequency fluctuations

Here, we consider the plasma response at frequencies close to the electron plasma frequency, ω_{pe}. In the collective regime ($k \ll k_{De}$) the maxima of the dynamic form factor correspond to resonances at the Langmuir wave frequencies. We will also consider transitional region into noncollective plasma response ($k \geqslant 0.5k_{De}$) at conditions of some plasma experiments. For high frequencies we can neglect ion dynamics, i.e., taking $J_i = J_N^R = g = 0$ in Eq. (51), and write the dynamic form factor as $S^{\rm HF}$,

$$S^{\rm HF}(\vec{k},\omega) = 2{\rm Re}\big[K_e^{\rm HF}(\vec{k},\omega)\big] \tag{55}$$

$$K_e^{\rm HF}(\vec{k},\omega) = \frac{i\big[1 + i\omega J_N^N\big(1 - k^2\lambda_{De}^2\big)\big]}{\omega\epsilon(\vec{k},\omega)k^2\lambda_{De}^2}, \tag{56}$$

where the plasma dielectric function Eq. (29) is taken in the following form:

$$\epsilon^{\rm HF}(k,\omega) = 1 + \frac{1 + i\omega J_N^N}{k^2\lambda_{De}^2}. \tag{57}$$

Only one moment, J_N^N, of the basis function ψ_0^{eN} Eq. (8) is required in the above expressions to properly describe the collisional plasma response at high frequencies.

In addition to the complete theory of the dynamical form factor Eq. (51) and its simpler version that is applicable to high frequency regime Eq. (55) we will also consider the simpler and commonly used model based on the Bhatnagar-Gross-Krook (BGK) [41,42] approximation to the collision operator. Our implementation of the BGK model involves the following expression for the plasma dielectric function,

$$\epsilon^{\rm BGK}(\vec{k},\omega) = 1 + \frac{4\pi e^2}{m_e k^2}\int d^3v \frac{1}{\omega + i\nu_{ei}(v) - \vec{k}\cdot\vec{v}}\vec{k}\cdot\frac{\partial F_M^e}{\partial\vec{v}}, \tag{58}$$

where the electron-ion collision frequency, $\nu_{ei} = 4\pi Z e^4 n_e \Lambda_{ei}/m_e^2 v^3$, is the velocity-dependent function. The $S(\vec{k},\omega)$ based on the BGK approximation Eq. (58) to particle collisions is derived from the fluctuation dissipation theorem [2,33],

$$S^{\rm BGK}(\vec{k},\omega) = -\frac{2k^2\lambda_D^2}{\omega}{\rm Im}\left[\frac{1}{\epsilon^{\rm BGK}(\vec{k},\omega)}\right]. \tag{59}$$

Figure 5 compares results of the collisionless theory, $S^V(\vec{k},\omega)$ Eq. (52) (dashed lines) or equivalently its simplified version of Ref. [8], complete theory of the dynamical form factor, Eq. (51) or (55) (continuous lines) and simplified theory of fluctuations in the collisional plasma based on the BGK model Eq. (59) (dotted lines marked with letters BGK). We show only red-shifted peak that is symmetric with the blue shifted feature. We have examined parameters of the experiment in Ref. [27] that were discussed for the low frequency response and shown in Fig. 3 ($n_e = 5.610^{18}$ cm^{-3}, $T_e = 100$ eV, $Z = 6$, $\alpha = 1.58$, $\theta = 117^\circ$). As we wrote before for these parameters of the experiment in Ref. [27] the electron collisional parameter, $k\lambda_{ei} = 133$, is very large and collisions do not affect the calculations. All three theories produce dynamic form factors that are indistinguishable. Therefore, to illustrate the effects of electron collisions we consider modified plasma parameters with electron temperature $T_e = 50$ eV and further introduce scattering angle $\theta = 10^\circ$ in Fig. 5(a). With these new parameters $\alpha = 21.8$ and $k\lambda_{ei} = 4.05$. For such a small value of $k\lambda_{De}$ collisionless theory Eq. (52) gives an extremely narrow and small peak. The plasma density fluctuations are defined entirely by collisional processes in spite of the weakly collisional regime at this wavelength of fluctuations. In Fig. 5(b) the scattering angle is $\theta = 60^\circ$ that gives $\alpha = 3.8$ and $k\lambda_{ei} = 23$. Even for these weakly collisional effects the complete theory significantly differs from the dominant collisionless peak and agrees reasonably well with the BGK approximation. The discrepancy between full kinetic model results Eq. (55) and the calculations based on the BGK approximation Eq. (59) underscores need for the careful modeling of collisions with inclusion of high angular harmonics, frequency effects, and electron-electron in addition to electron-ion collisions. Perhaps the most surprising outcome of the comparisons in Fig. 5 corresponds to the essentially collisionless regime in Fig. 5(b) that still displays significant deviations from the collisionless theory at the large values of

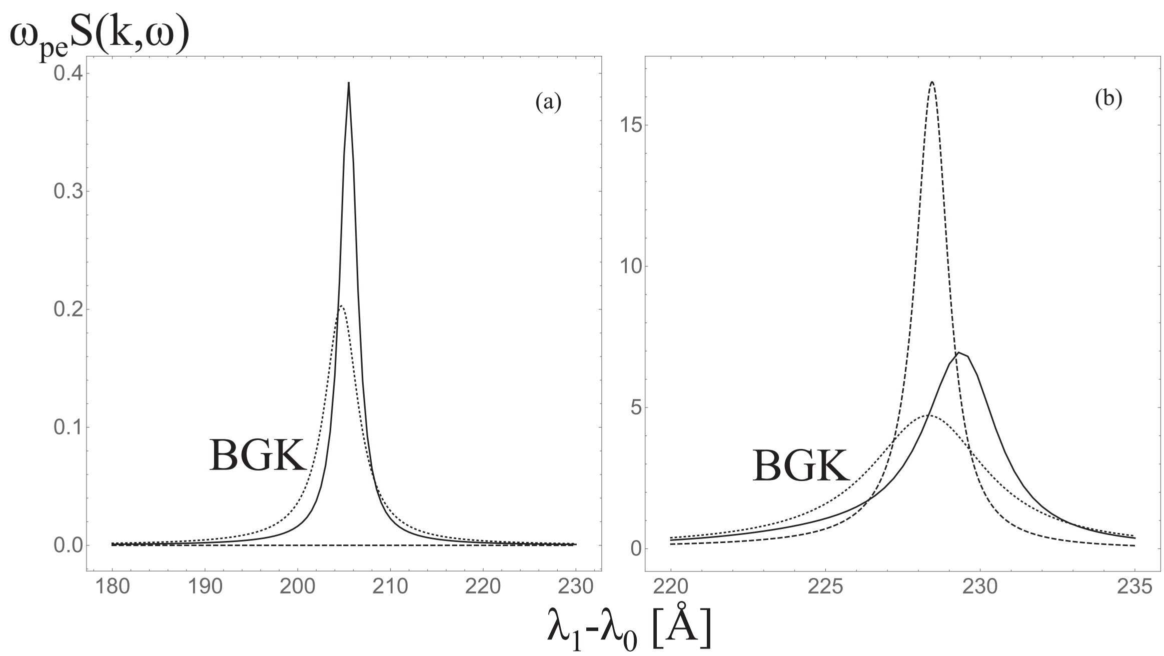

FIG. 5. Dynamic form factors for $n_e = 5.6\ 10^{18}$ cm^{-3}, $\lambda_0 = 5270$, $T_e = 50$ eV. (a) $\theta = 10^\circ$, (b) $\theta = 60^\circ$. Dashed lines correspond to the collisionless RPA theory Eq. (52), continuous lines correspond to the complete theory of the dynamic form factor, Eq. (51) or (55) and the BGK model Eq. (59) is shown by dotted lines, marked with BGK.

the collisionality parameter $k\lambda_{ei} = 23$. Collisions in the BGK model or in the complete theory are responsible for 30% to 50% differences in the amplitude of the Langmuir waves peaks.

Effect of collisions on the plasma dispersion function in the high frequency regime and the modifications of the Langmuir wave dispersion relation have been well understood (cf., e.g., Ref. [43]). Also our solutions to the kinetic equations (see Sec. II) and in particular the method of harmonics summation by the renormalized effective collision frequency Eq. (10) have been applied before to the description of Langmuir wave dispersion and damping [18,44]. In particular, Ref. [44] examined the BGK approximation Eq. (58) and compared it with the approach involving harmonic expansion and effective collision frequency Eq. (10). Discrepancies between dielectric functions derived from the rigorous solution of the full kinetic equation of Sec. II and from the approximate models such as the BGK operator lead also to differences in the dynamical form factors.

The most interesting developments in TS in recent years has been application of short wavelength probes, from the VUV to hard x-ray parts of the spectrum in experiments with dense highly compressed plasmas [4]. An example in Fig. 6 illustrates results for the parameters of such experiment [45], where an x-ray beam (photon energy 5.5 keV) from a free electron laser is scattered from cryogenic hydrogen. This example is focused on scattering in the high-frequency regime because the spectral resolution of the scattered light that is used in these experiments does not yet allow measurements of the separation of ion acoustic peaks in the low-frequency regime. Figure 6 shows again red-shifted Langmuir wave peaks for the parameters of experiment with the x-ray probe, $\lambda_0 = 2.25$ Å scattered from liquid hydrogen jet at $n_e = 10^{23}$ cm^{-3} and $T_e = 20$ eV at $\theta = 10^\circ$. At these parameters the plasma is approaching the strongly coupled regime and in spite of the very short probe wavelength the fluctuations are also affected by collisions, $k\lambda_{ei} = 1.4$. Also for $\alpha = 1/k\lambda_{De} = 1.97$ this is

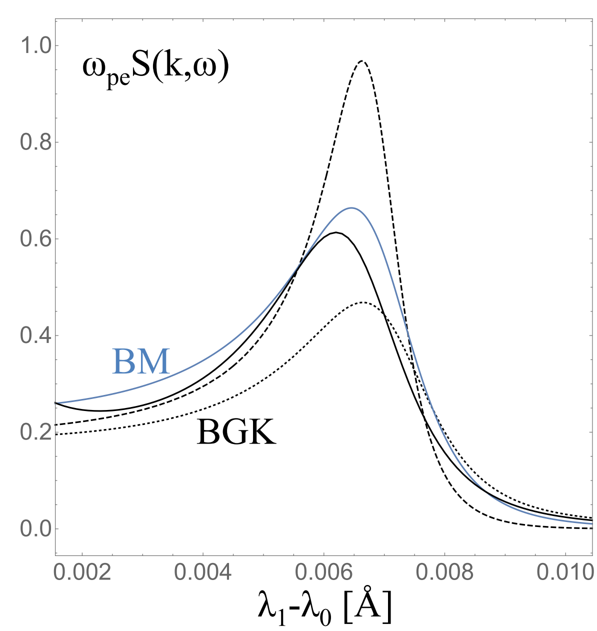

FIG. 6. Dynamic form factors in the high frequency regime corresponding to H plasmas from x-ray probe scattering experiment with $\lambda_0 = 2.25$ Å, $n_e = 10^{23}$ cm^{-3}, $T_e = 20$ eV, $\theta = 10^\circ$, $\alpha = 1.97$. Dashed lines correspond to the collisionless theory Eq. (52), continuous lines correspond to the complete theory of the dynamical form factor, Eq. (51) or (55) and the BGK model Eq. (59) is shown by dotted lines. For comparison, we also show results from the Born-Mermin (BM) approximation [46].

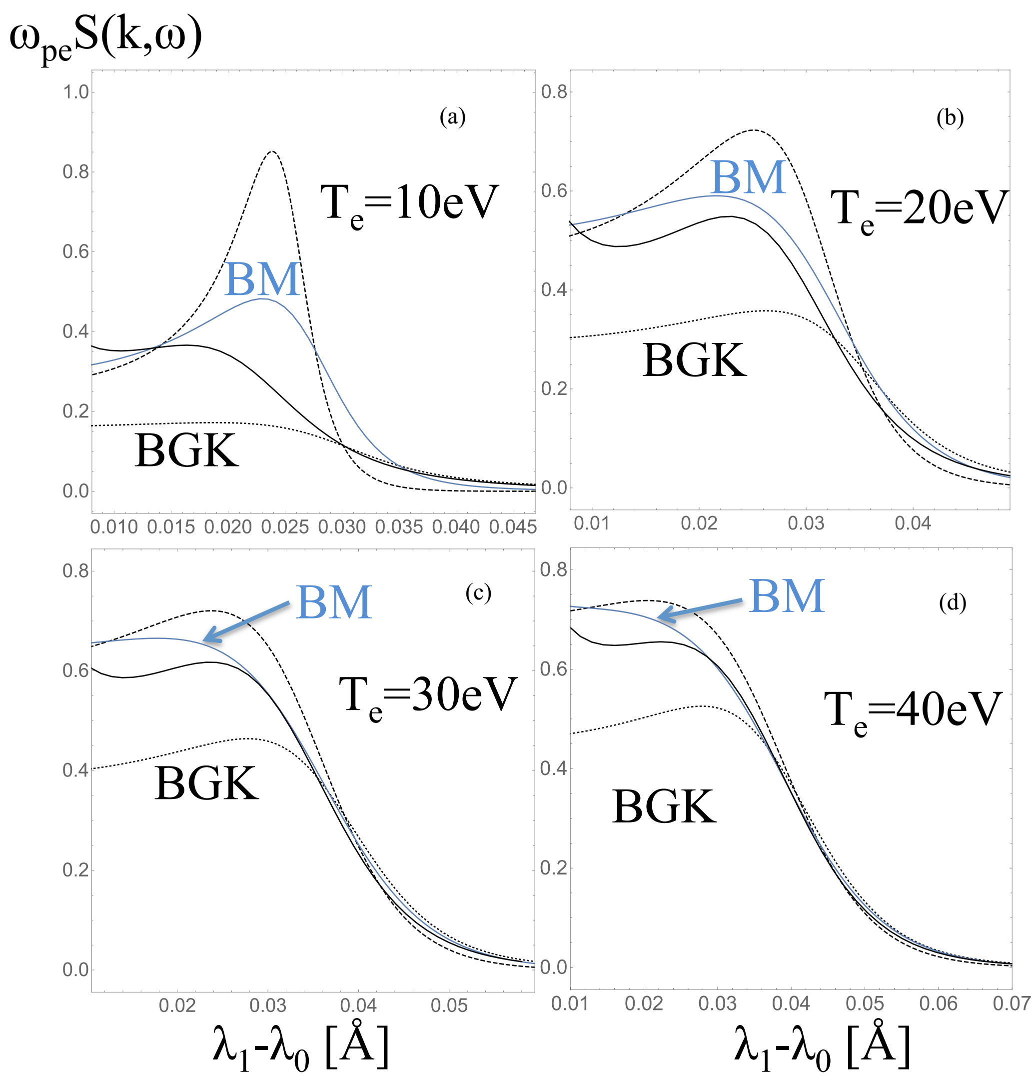

FIG. 7. Dynamic form factors in the high frequency regime, red shifted peak, corresponding to Be ($Z = 2, A = 9$) plasmas from x-ray scattering experiments with $\lambda_0 = 4.19$ Å, $n_e = 10^{23}$ cm^{-3}, $\theta = 30°$. Dashed lines correspond to the collisionless theory Eq. (52), continuous lines correspond to the complete theory of the dynamical form factor, Eq. (51) or (55) and the BGK model Eq. (59) is shown by dotted lines. (a) $T_e = 10$ eV, $k\lambda_{ei} = 0.28$, $\alpha = 1.74$; (b) $T_e = 20$ eV, $k\lambda_{ei} = 1.12$, $\alpha = 1.23$; (c) $T_e = 30$ eV, $k\lambda_{ei} = 2.51$, $\alpha = 1$; (d) $T_e = 40$ eV, $k\lambda_{ei} = 4.46$, $\alpha = 0.87$; For comparison, we also show results from the BM approximation [46].

the transitional regime to noncollective plasma response and at $Z = 1$ higher-order corrections due to e-e collisions are comparable to e-i collision contributions. While evaluating collisional form factors Eq. (51) or (55) and $S^{\rm BGK}$ Eq. (59) in Fig. 6, we have clamped Coulomb logarithms at 2 in the collisional frequencies. This approximate treatment of the strongly coupled limit will be evaluated by comparison with theory that includes proper treatment of collision frequency in dense plasmas, such as the Born-Mermin (BM) approximation [4,46–48]. The BM approximation employs the Mermin expression [38,39] for the electronic part of the susceptibility function,

$$\chi_e^{\rm BM}(k,\omega) = \frac{\left(1 + i\frac{\nu^B(\omega)}{\omega}\right)\chi_e^{\rm RPA}[k,\omega + i\nu^B(\omega)]}{1 + i\frac{\nu^B(\omega)}{\omega}\frac{\chi_e^{\rm RPA}(k,\omega+i\nu^B(\omega))}{\chi_e^{\rm RPA}(k,0)}}, \qquad (60)$$

where $\nu^B(\omega)$ is the elecron-ion collision frequency and $\chi_e^{\rm RPA} = \chi_e^V$ in classical plasmas. In BM theory, $\nu^B(\omega)$ is approximated by the second-order perturbation expansion in terms of the electron–ion interaction (Born approximation) [49],

$$\nu^{\rm B}(\omega) = -i\frac{\epsilon_0 Z_f}{6\pi^2 e^2 m_{\rm e}}\int_0^\infty dq\, q^6\left[V_{ei}^S(q)\right]^2 S_{ii}(q)\frac{1}{\omega} \times\left[\chi_e^{\rm RPA}(q,\omega) - \chi_e^{\rm RPA}(q,0)\right]. \qquad (61)$$

$V_{\rm ei}^S(q) = e^2/\epsilon_0 q^2(1 + \kappa_{\rm sc}^2/q^2)$ is the statically screened electron–ion potential with the inverse screening length $\kappa_{\rm sc}$, $S_{\rm ii}(q)$ is the static ion–ion structure factor, taken here in the Debye-Hückel approximation $S_{\rm ii}(k) = k^2/(k^2 + \kappa_D^2)$. The dynamical form factor $S^{\rm BM}(\vec{k},\omega)$ in BM approximation is evaluated using Fluctuation-Dissipation theorem Eq. (59), where $1 + \chi_e^{\rm BM}$ Eq. (60) replaces $\epsilon^{\rm BGK}$. $S^{\rm BM}(\vec{k},\omega)$ is shown by the blue solid line in Fig. 6.

The BM theory accounts for electron–ion collisions only. Replacing $\chi_e^{\mathrm{RPA}}(k,\omega)$ in Eq. (60) by an expression for the correlated electron-electron susceptibility, using the local field correction (LFC) leads to the generalized BM approximation describing also electron-electron collisions [50,51]. The BM curve in Fig. 6 describes collisional broadening of electron-plasma resonance in qualitative agreement with the other collisional theories. The peak position is closer to the collisionless RPA result than the other theories. This is a well known and understood feature of the BM approximation, which due to the analytical structure of the BM theory can only describe the plasmon dispersion of the RPA. Deviations from the RPA dispersion have been observed in the generalized BM approximation [50]. However, for the present plasma parameters, also the generalized BM approximation does not yield a significant deviation from the RPA dispersion.

The final example is related to first collective x-ray scattering measurements of fluctuations at plasma frequency in beryllium solid density plasmas [52] as illustrated in Fig. 7. We have examined plasmas with increasing electron temperatures, while all other parameters (scattering angle, plasma density, and photon energy) are kept constant. Figures 7(b), 7(c) and 7(d) show form factors in regimes where plasma response become gradually more noncollective, $\alpha = 1.23$, 1, 0.87, respectively. All theories, S^V Eq. (52), S^{BM} Eq. (60), and S^{HF} Eq. (55) converge to a single curve with exception of the BGK approximation Eq. (59) that greatly overestimates the effect of collisions. Comparisons between form factors at $T_e = 10\,\mathrm{eV}$, Fig. 7(a), show strong effect of collisions in S^{HF} Eq. (55) and lesser modifications by *e-i* collisions in the BM Eq. (60) results. The collisionless theory S^V Eq. (52) and BGK approximation Eq. (59) are the least accurate. The difference between BM approximation and the S^{HF} Eq. (55) demonstrate importance of the proper modeling of the collision frequency. The simple fix of clamping Coulomb logarithms at two that is used in S^{HF} Eq. (55) to account for the strongly coupled plasma effects overestimates importance of collisions in Fig. 7(a) as compared with BM results.

V. SUMMARY AND CONCLUSIONS

The theory of density fluctuations of this paper is firmly based on the results of nonlocal and nonstationary plasma hydrodynamics which has been developed over the years [16–18,21,22] and gives rigorous solutions to the linearized Vlasov-Landau kinetic equation for the two component plasmas. This approach is well suited to the problem of evaluating plasma fluctuations and the dynamic form factor because solutions for the small perturbations of the particle distribution functions are found in terms of Fourier transforms in space and time. The background state is described by Maxwellian distribution functions with two different temperatures. We have derived a new expression for $S(\vec{k},\omega)$ for nonequilibrium two temperature plasmas. We also verified that the proper expression in the collisionless limit is recovered.

The main result of our theory of the dynamic form factor $S(\vec{k},\omega)$ Eq. (51) is applicable to TS experiments in the entire weakly coupled plasma regime from the hydrodynamical fluctuation limit modelled by Braginskii equations to the collisionless RPA limit of Eq. (52). Our theory works for nonequilibrium plasmas with different temperatures of electrons and ions, $T_e \neq T_i$. Therefore, it avoids limitations imposed by the application of the fluctuation dissipation theorem [2,33] in calculations of the $S(\vec{k},\omega)$ that requires a complete equilibrium state to be valid. From the analysis of several TS experiments and by studying examples corresponding to laboratory plasmas in Sec. IV we have found that collisions matter and can change the form factor more than experimental accuracy of the fits. This is true even for the collisional parameter, $k\lambda_{ab} \geqslant 10$. Such plasmas are in the weakly collisional regime that is well described by our nonlocal and nonstationary hydrodynamics. We expect that with time our program using Mathematica to evaluate Eq. (51) will be commonly available and our theory will be widely used to reproduce experimental TS data.

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